

A comment on technical naturalness and the cosmological constant

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ABSTRACT: We propose a model of dynamical relaxation of the cosmological constant. Technical naturalness of the model and the present value of the vacuum energy density imply an upper bound on the supersymmetry breaking scale and the reheating temperature at the TeV scale.

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From an effective field theory perspective the problem with the cosmological constant term is that it is a relevant term that seems to be small for no apparent reason. In supersymmetric theories the cosmological constant is of the order of M_{SUSY}^4 (where M_{SUSY} is the supersymmetry breaking scale) which is at least 10^{-60} in Planck units, while the actual value of the vacuum energy density is only about 10^{-120} . The growing evidence for inflation (for recent results that support inflation see [1]) makes the problem even more intriguing since inflation is driven by vacuum energy much larger than 10^{-120} . Accordingly, the natural question is: why is the ratio of the vacuum energy during inflation to the current vacuum energy so large (and yet not infinite)?

At least conceptually the simplest resolution to this question is that there is a dynamical adjustment mechanism that ensures the smallness of the vacuum energy at the end of inflation. Assuming such a mechanism can be described by a low energy effective action the challenge is to come up with an effective field theory that appropriately adjusts the vacuum energy and is natural, or at least technically natural. More than twenty years ago Abbott proposed a technically natural model that does exactly that [2]. However, as pointed out by Abbott himself, the problem with his model is that the vacuum energy is adjusted via tunnelling from one local minimum to the next. Hence, much like in old inflation [3], the resulting universe is empty, containing neither radiation nor matter.

In this note we attempt to improve on Abbott's model by proposing a model that evades the emptiness problem and appears to be technically natural.¹ The effective action we propose takes the form (we work in units where $M_{\text{Pl}} = (8\pi G)^{-1/2} = 1$)

$$S = S_{\text{EH}} + S_{\text{relaxation}} + S_{\text{inflation}}. \tag{1}$$

S_{EH} is the Einstein-Hilbert action and the relaxation action is

$$S_{\text{relaxation}} = \int d^4x \sqrt{-g} \left(-\frac{1}{2}(\partial\psi)^2 - V_{\text{ren}} - V(\psi) \right). \tag{2}$$

V_{ren} is the renormalized value of the cosmological constant including radiative contributions from all fields. We assume that V_{ren} is large and positive. The role of $S_{\text{relaxation}}$ is to reduce the vacuum energy slowly in a controlled fashion. For this reason we follow [2] (see also [4, 5]) and take

$$V(\psi) = \epsilon \psi \quad \text{where} \quad |\epsilon| \ll 1.$$

The condition $|\epsilon| \ll 1$ is essential for the model and must not receive large quantum corrections. Since $\epsilon\psi$ is a relevant term one might suspect that small ϵ is as unnatural as a small cosmological constant. This, however, is not the case since $\epsilon\phi$, unlike the cosmological constant, is protected by a symmetry: when $\epsilon = 0$ the action is invariant under the shift symmetry $\psi \rightarrow \psi + \text{const.}$ which ensures that corrections to ϵ are proportional to ϵ . Namely $|\epsilon| \ll 1$ is technically natural.

As a result of this small slope ψ will roll down the potential reducing the vacuum energy. The slow roll approximation is valid as long as the vacuum energy is much larger than ϵ . As argued above we can take ϵ to be arbitrarily small so the slow roll approximation

¹A different approach to the emptiness problem was suggested recently in [6].

is valid at arbitrarily small vacuum energy. This way we are able to reduce the vacuum energy but not to convert it into kinetic energy, which is necessary to avoid the emptiness problem.

The role of $S_{\text{inflation}}$ is to fix that while making sure that the vacuum energy at the end of inflation is small. There are various actions one might wish to consider. Below we study the one that we believe is the simplest single-field $S_{\text{inflation}}$ that illustrates how the adjustment mechanism works. We do not address phenomenological aspects of the model nor possible realization in string theory. These issues are likely to require a more complicated multi-field action. The action takes the form

$$S_{\text{EH}} + S_{\text{inflation}} = \int d^4x \sqrt{-g} \left[\frac{1}{2} e^{-\phi^2} R - V(\phi) \right] + \int d^4x \mathcal{L}_{\text{kin}}. \quad (3)$$

The first term is familiar from supergravity theories before rescaling to the Einstein frame. \mathcal{L}_{kin} is a bit complicated in this frame and is fixed by requiring that in the Einstein frame it takes the standard form (see (5)). The details of \mathcal{L}_{kin} (that can be found for example in chapter 21 of [7]) will play no role below. $V(\phi)$ is designed to have the following properties. It has a maximum at $\phi = 0$

$$\gamma \equiv - \left. \frac{d^2 V(\phi)}{d\phi^2} \right|_{\phi=0} > 0,$$

and the difference between its values at the maximum and the minimum is

$$\Delta V \equiv V_{\text{max}} - V_{\text{min}} = \frac{\gamma}{4}. \quad (4)$$

$S_{\text{inflation}}$ modifies the dynamics in the following way. Expanding the first term in (3) we see that the effective mass of ϕ is $m_{\text{eff}}^2 = R - \gamma$. In the slow roll approximation the relationship between the curvature and the vacuum energy density is $R = 4V$. Therefore, for $V > \gamma/4$ we have $m_{\text{eff}}^2 > 0$ and ϕ does not receive an expectation value and does not affect the FRW equations that are controlled by ψ . When the vacuum energy density crosses the critical value $V_c = \gamma/4$ an instability is developed and ϕ acquires an expectation value. This reduces R which further reduces m_{eff}^2 which in turn increases the expectation value of ϕ even further. Eq. (4) ensures that the end result of this feedback mechanism is a space with no vacuum energy at all. This follows from the fact that for $R = 0$ the effective potential for ϕ is simply $V(\phi)$ and so the vacuum energy that is lost due to the condensation of ϕ is ΔV which (from (4)) cancels V_c exactly.

It is useful to see how this comes about in the Einstein frame, where the action is

$$S_{\text{EH}} + S_{\text{inflation}} = \int d^4x \sqrt{-g} \left[\frac{1}{2} R - 3(\partial\phi)^2 - \tilde{V} \right], \quad (5)$$

with

$$\tilde{V} = e^{2\phi^2} (V(\phi) + V_{\text{ren}} + \epsilon\psi). \quad (6)$$

Thus m_{eff}^2 depends on the total vacuum energy $m_{\text{eff}}^2 = -\gamma + 4(V(\phi = 0) + V_{\text{ren}} + \epsilon\psi)$, which implies that an instability is developed at $V = V_c = \gamma/4$ and that (since $\Delta\tilde{V} = \Delta V$) the condition for the cancelation of the vacuum energy is indeed (4).

Two clarifications are in order. First, the cancelation of the vacuum energy described above does not involve fine tuning V_{ren} . In fact the only assumption we made is that $V_{\text{ren}} + V(\phi = 0) > V_c$. Second, the feedback mechanism described above depends only on $V(\phi)$ and not on ϵ . Thus ϵ can be taken to be small so that on time scales of the age of the universe the vacuum energy density does not change significantly.² This ensures that the present value of the vacuum energy is of the order of its value at the end of inflation.

The advantage of this model over [2] is that now, much like in new inflation [8, 9], ϕ has plenty of kinetic energy that can heat the universe. There are a couple of general bounds on the reheating temperature, T_r . First, energy conservation implies that $T_r^4 \leq V_c$ where we ignore factors of order one and the number of fields. Second, potential energy is converted into kinetic energy only when the slow roll approximation breaks down. This implies that $T_r^4 < \gamma$. In our case $V_c \sim \gamma$ and so

$$T_r \leq V_c^{1/4}. \tag{7}$$

To saturate this bound in the simplest scenario of reheating via ϕ decay [12, 13] the decay rate should be $\Gamma_\phi \sim m_\phi \sim \gamma^{1/2}$.

The discussion so far has not included quantum corrections to $V(\phi)$ and as a result (7) does not imply a relation between T_r and the present value of the vacuum energy density. In what follows, we show how quantum effects and the present value of the vacuum energy density impose an upper bound on T_r . For concreteness we consider the following potential

$$V(\phi) = -\frac{1}{2}m^2\phi^2 + \frac{1}{4}g\phi^4. \tag{8}$$

The condition for the cancelation of the vacuum energy, (4), in this case is

$$m^2 = g. \tag{9}$$

Quantum corrections to (9) and more generally to the effective potential, $V_{\text{eff}}(\phi)$, yield a non-vanishing vacuum energy at the end of inflation that we denote by V_0 . Technical naturalness of the model requires that $V_0 \sim 10^{-120}$. This is hard to achieve in non-supersymmetric theories since the potential contains a relevant term that receives large quantum corrections. In supersymmetric theories these corrections are suppressed and (ignoring logarithmic factors) scale like

$$V_0 \sim gm^2 \sim m^4. \tag{10}$$

Thus technical naturalness of the vacuum energy implies that $m \sim 10^{-30}$ and that the upper bound on T_r is approximately $m^{1/2} \sim 10^{-15} \sim 1$ TeV.

Note that $m \sim 10^{-30}$ implies a fifth force deviation from GR at scales of the order of $1/m \sim 100$ microns since the Newton constant depends on the expectation value of

²Non-perturbative effects such as [10, 11] impose a lower bound on ϵ . Since these are exponentially small this bound does not matter on time scales of the age of the universe. Moreover, in multi-field generalizations of this model one can avoid these effects.

ϕ .³ This prediction is particularly interesting in light of current experiments [18, 19] that should be able to detect such a deviation in the near future.

When supersymmetry is broken these corrections are enhanced. In particular

$$\delta m_\phi^2 \sim g \Delta m^2, \quad \text{where} \quad \Delta m^2 = m_\phi^2 - m_{\tilde{\phi}}^2. \quad (11)$$

δm_ϕ^2 represents the quantum corrections to m_ϕ^2 and $m_{\tilde{\phi}}$ is the mass of the super-partner of ϕ . Technical naturalness implies that

$$\delta m_\phi^2 \leq V_0. \quad (12)$$

Thus Δm^2 is at most of the order of 10^{-60} and the upper bound on the supersymmetry breaking scale in the ϕ sector is $M_{\text{SUSY}}^\phi \sim 10^{-30}$. Hence ϕ cannot be a field in the MSSM (or any other supersymmetric generalization of the standard model) or in the hidden sector responsible for the SUSY breaking. In fact such a low SUSY breaking scale in the ϕ sector implies an upper bound on M_{SUSY} at around 1 TeV. This follows from the fact that gravity always mediates SUSY breaking from one sector to the other, which in our case gives

$$M_{\text{SUSY}}^\phi \sim M_{\text{SUSY}}^2 \quad \Rightarrow \quad M_{\text{SUSY}} \sim \sqrt{M_{\text{SUSY}}^\phi} \sim 10^{-15} \sim 1 \text{ TeV}. \quad (13)$$

Coupling with gravity is another source for quantum corrections to the potential. It is easier to estimate these effects in the Einstein frame (5). When $\tilde{V} = 0$ there is a shift symmetry, $\phi \rightarrow \phi + \text{const.}$ that prevents gravity loops from generating a potential for ϕ . Hence corrections to \tilde{V} due to gravity loops are proportional to \tilde{V} and on dimensional grounds the one loop contribution scales like $\delta \tilde{V} \sim \tilde{V} \Lambda^2$, where Λ is the cutoff scale. In supergravity theories we expect the gravitino mass, $m_{3/2}$, to play the role of Λ . Since $m_{3/2} \sim M_{\text{SUSY}}^2 \sim m_\phi$ and $V_c \sim m_\phi^2$ we have

$$\delta V_{\text{eff}} \sim M_{\text{SUSY}}^8, \quad (14)$$

which leads to the same upper bound on M_{SUSY} at around 1 TeV.

Alternatively, we may reach the same conclusion by applying similar reasoning to the Kahler potential. Coupling with gravity can generate corrections to the Kahler potential of the form $|\phi|^{2n}$ with $n > 1$. When $\tilde{V} = 0$, the shift symmetry implies that these corrections should vanish because otherwise they would lead to terms of the form $|\partial\phi|^2 |\phi|^{2n-2}$ that do not respect the shift symmetry. Thus, gravity corrections to the Kahler potential scale like \tilde{V} and their effects on the effective potential scale like $\tilde{V}^2 \sim M_{\text{SUSY}}^8$.⁴

Another quantum gravity effect that should be considered is due to higher order terms, such as R^2 . These will modify the FRW equations and as a result will shift V_c . This effect scales like V_c^2 and since $V_c \sim 10^{-60}$ it will not spoil the naturalness of the vacuum energy.

The model described above relies heavily on low scale SUSY and it is natural to ask whether there are other models that are technically natural with high-scale SUSY or maybe

³A different approach to the cosmological constant problem that leads to a similar prediction is known as fat gravity [15–17].

⁴For example the term $V(\phi) = m^2 |\phi|^2$ can generate a correction to the Kahler potential that scales like $m^2 |\phi|^4$ which in turn will generate $\delta V \sim (m^2 |\phi|^2)^2$.

even with no SUSY at all. Below we describe a non supersymmetric model that “almost” does that and fails to be technically natural only because of gravity loop effects.

Consider the following action

$$S_{\text{EH}} + S_{\text{inflation}} = \int d^4x \sqrt{-g} \left[\frac{1}{2} e^{-\phi^2} R - \frac{1}{2} (\partial\phi)^2 + \frac{1}{16\pi^2} \left(\frac{\phi}{f} - \pi \right) \text{Tr}(F \wedge F) \right] . \quad (15)$$

The shift by π is needed for $\phi = 0$ to be a maximum of the potential induced by the instantons. If the gauge theory has a weakly coupled fixed point (see e.g. [14]) then $V(\phi)$ is dominated by the one-instanton contribution that takes the form

$$V(\phi) = V_g \cos\left(\frac{\phi}{f}\right), \quad \text{with} \quad V_g = C e^{-8\pi^2/g^2}. \quad (16)$$

C is a constant that depends on the details of the UV cutoff on the integration over the size of the instanton, which we assume to be at the Planck scale. Moreover, C receives large quantum corrections because quantum effects shift $1/g^2 \rightarrow 1/g^2 + \text{const.}$ in the exponent. Thus without fine tuning only the order of magnitude of V_g can be fixed.

The nice feature of this model is that V_g drops out of the condition for the cancelation of the vacuum energy, (4), that takes the form

$$f^2 = \frac{1}{8}. \quad (17)$$

The fact that g is not exponentially small does not lead to large corrections to (17) since as argued above loops around the one instanton background can modify V_g but not f . The leading corrections to (17) come from two-instanton effects that scale like V_g^2 . Thus naturalness of the vacuum energy implies that $V_g \sim 10^{-60}$ and as before the upper bound on T_r is at the TeV scale.

Unfortunately, gravity loops spoil the naturalness of the model. A minimal coupling of an axion with gravity respects the shift symmetry

$$\phi \rightarrow \phi + 2\pi n f \quad (18)$$

of (16) and thus can modify V_g but not (17). However, ϕ does not couple minimally with gravity. The coupling $e^{-\phi^2} R$ breaks (18) and generates terms in the effective potential that do not respect (18) and (without low scale SUSY) spoil the technical naturalness of the model.

It is interesting that in this model only gravity loops are problematic, while the cosmological constant problem is usually associated with field theory loops. To be more precise, normally, treating gravity semi-classically

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \langle T_{\mu\nu} \rangle, \quad (19)$$

does not help much with the cosmological constant problem since field theory loops still induce a large cosmological constant term that affects the geometry via the semi-classical equation (19). The amusing aspect of the axion model (15) is that if one (wrongly) assumes (19) then a small vacuum energy density is technically natural even without SUSY.

We of course would like to consider gravity at the quantum level. Hence if we do not want to rely on low scale SUSY breaking then we have to come up with an alternative way to suppress the gravitational corrections to $V_{\text{eff}}(\phi)$. One possible way to do this is to replace the $R\phi^2$ trigger by a trigger that depends on the Gauss-Bonnet combination $R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$. The topological nature of the Gauss-Bonnet term suppresses corrections to $V_{\text{eff}}(\phi)$ even without SUSY.⁵ However, in the slow-roll approximation the Gauss-Bonnet term scales like V^2 and not like V . This complicates the model and (as far as we can tell at the moment) lowers considerably the upper bound on T_r .

To summarize, the goal of this note was to suggest a simple mechanism that renders the small value of the vacuum energy density technically natural. The mechanism was illustrated in the context of a concrete example that led to the following predictions:

- (i) The upper bound on the reheating temperature is about 1 TeV.
- (ii) SUSY is broken at around the TeV scale.
- (iii) There is a fifth force deviation from GR at scales of the order of 100 microns.

We believe that it is reasonable to suspect that (ii) and (iii) could be relaxed in generalizations of this model. It is, however, hard to see how models that use the same basic mechanism could lead to T_r much larger than 1 TeV. As usual in models with low scale inflation this makes the issue of baryogenesis subtle because the electroweak scale is not too far below 1 TeV. In our case this issue is even more subtle since the interactions that transfers the kinetic energy of ϕ into heat (and eventually to baryons) should on the one hand be strong enough to yield $T_r \sim 1$ TeV (or at least to avoid the cosmological moduli problem associated with a particle with such a small mass) and on the other hand it should not spoil the technical naturalness of the model. This implies that this interaction should involve only derivatives of ϕ since interaction that involves ϕ itself and gives $T_r \sim 1$ TeV will generate large quantum corrections to $V_{\text{eff}}(\phi)$.⁶ Even with interactions that involve only derivatives of ϕ one needs to make sure that subleading corrections to $V_{\text{eff}}(\phi)$ are suppressed. The symmetry (18) suggests that the axion model is less sensitive to such corrections than the ϕ^4 model.

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⁵For example the term $\lambda(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho})\phi^2$ leads to δm^2 that scales only like $\lambda^2\Lambda^{10}$.

⁶There are also finite temperature corrections to $V_{\text{eff}}(\phi)$. These, however, become negligible as the universe cools down and so they do not modify the present value of the vacuum energy.

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